Genetic network complexity: weights matter more than topology

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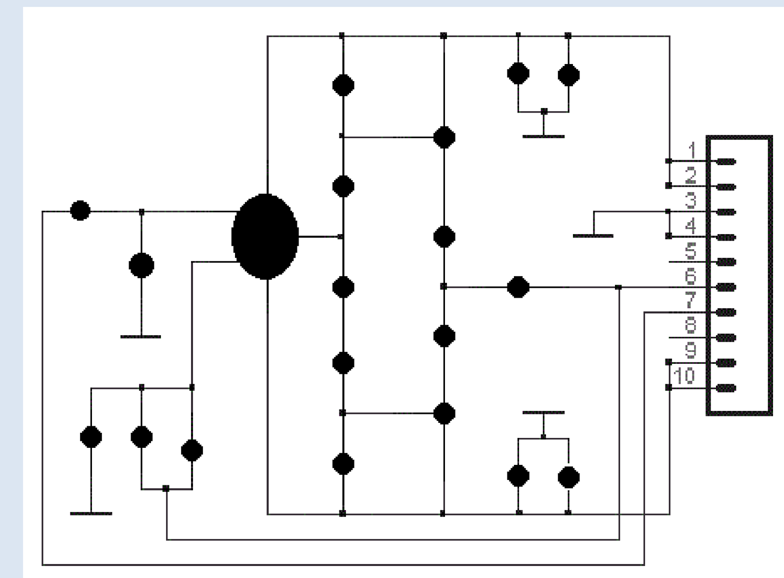
1. Appropriate level of description?

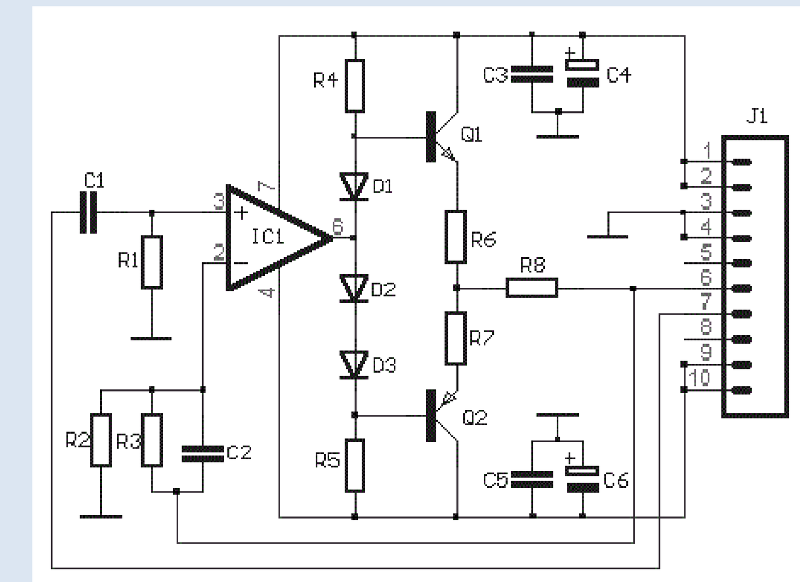
Can we understand genetic networks like we understand electronic circuits? Not every detail matters, but knowing only topology is insufficient.

Goal: build a toy model where the appropriate level of description can be constructed explicitly. How much do microscopic quantitative details matter?



Just topology: too coarse

⊂



Only relevant detail

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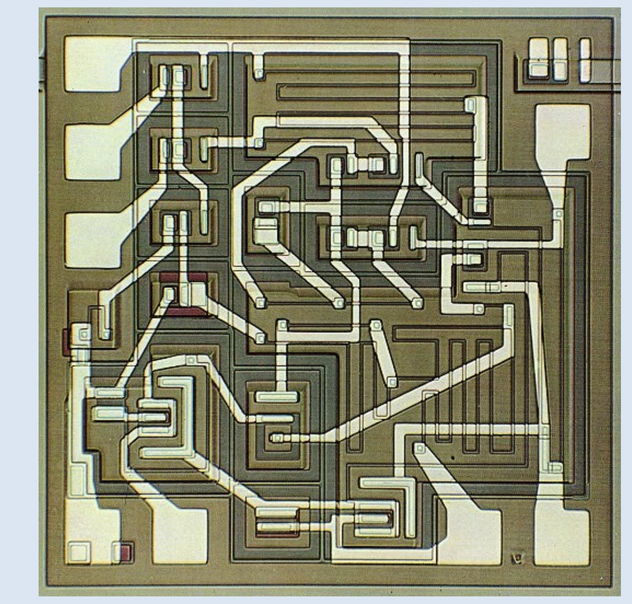
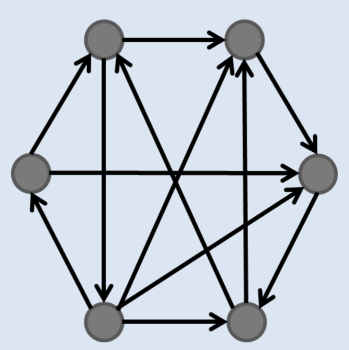


Too microscopic

2. Model: weighted graphs and complexity

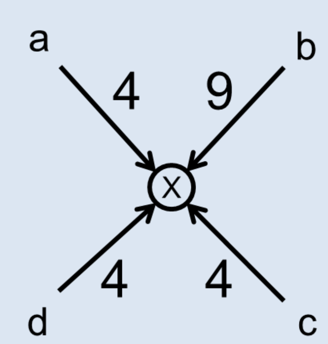


Genes: binary variables; interactions have variable strength.
A gene is activated if its inputs exceed a threshold:

$$s_i = \mathrm{sgn}\left(\sum_{j\to i} J_{ij} s_j + H_i\right) \qquad (1)$$

Capacity of a network: number of solutions (= number of cell types it can encode)

Each node implements a Boolean function from a *finite* set with a *non-arbitrary measure*.



⟺ $x = f(a,b,c,d)$

For a given topology, we can enumerate all of its *non-equivalent* and *equiprobable* (!) realizations as a weighted graph.

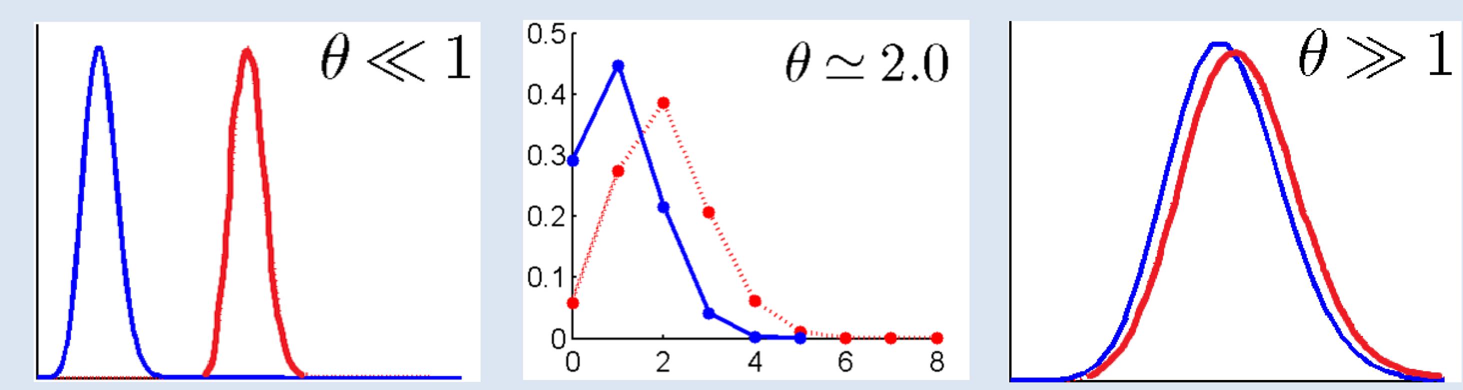
Define *complexity* as the diversity of possible causal relations in the graph.

Links are satisfied or frustrated. Define an *active* link as a link whose satisfied state is essential for equation (1) to hold. Each solution defines a binary sequence: the pattern of active links. Define diversity of a set of sequences as the length of the shortest path connecting all of them on a hypercube (traveling salesman).

How is complexity affected by the choice of topology vs. the choice of weights?

3. Conclusions: weights matter more

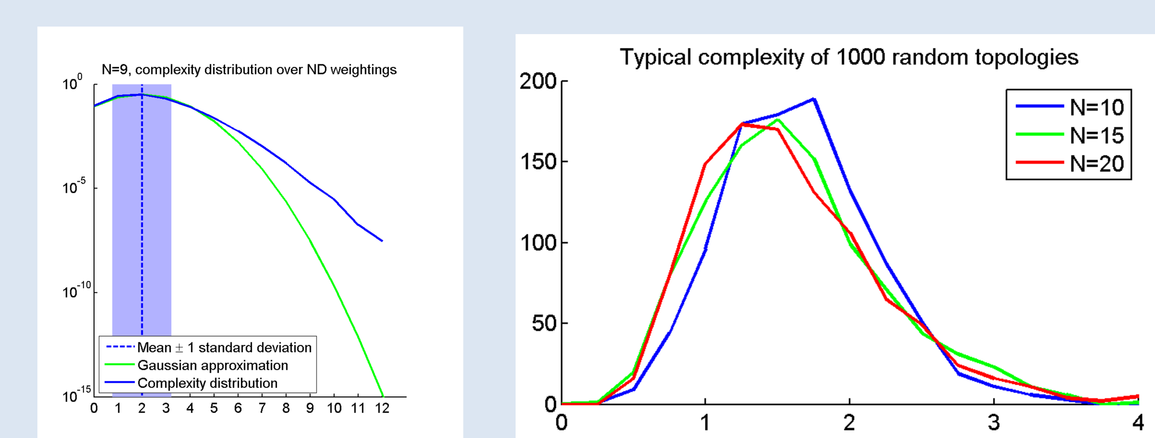
1. Relative importance of weights vs. topology is of order 1.



Capacity distributions over all choices of weights for 2 fixed topologies.
Left/right: capacity determined by topology/weights. Actual data is in the middle.
θ defined as $[\mathrm{mean}_{topology}(\sigma_{weights})]/[\sigma_{topology}(\mathrm{mean}_{weights})]$, σ is standard deviation

2. Optimal weights outperform optimal topology
(for 85% of topologies with N≤10)

3. Larger networks are not automatically more complex



Capacity distributions, both over weights and over topologies, are heavy-tailed. Typical capacity (random choice of weights) does not scale with network size.

For an information-processing network:

1. Topology and weights have effects of the same order.

2. High-complexity graphs operate in a non-generic parameter regime.

3. Evolving weights is a better strategy than changing topology / adding nodes

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