The Sum Capacity of K user Cognitive Interference Channel to within Constant Gap

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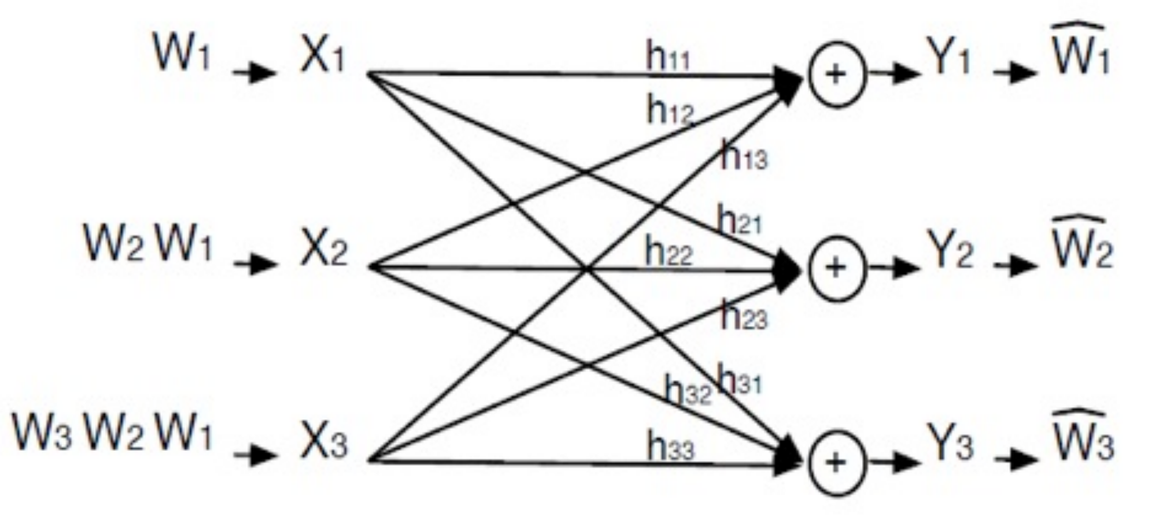
Abstract

We consider the K-user cognitive interference channel with one primary and K – 1 secondary/cognitive transmitters with a *cumulative message sharing* structure, i.e., cognitive transmitter i, i ∈ [2 : K], non-causally knows all messages of the users with index less than i. We first propose a computable outer bound valid for any memoryless channel and show the sum-rate to be achievable for the symmetric K- user Linear Deterministic Channel. For the K- user channel having only the K-th transmitter know all other messages is sufficient to achieve the sum-capacity, i.e., cognition at transmitters 2 to K−1 is not needed. Next, the sum-capacity of the symmetric Gaussian noise channel is characterized to within a constant gap, which depend on K. Moreover it is only required for transmitters 2 to K – 1 to have, in addition to their own message, non-causal message knowledge of the transmitter 1's message.

Channel Model



Past Work

In [1], several types of 3- user cognitive interference channels are proposed: that with "cumulative message sharing" (CMS) as considered here, that with "primary message sharing" (PMS) where the message of the single primary user is known at both cognitive transmitters

In [2] the sum-capacity for the 3-user channel with CMS for the linear deterministic channel (LDC) is obtained

In [3] the channel model consists of one primary user and K – 1 parallel cognitive users; each cognitive user only knows the primary message in addition to their own message (thus not a cumulative message structure); for this channel model the capacity in the "very strong" interference regime is obtained

Contributions

1) We obtain a novel outer bound region that reduces to the outer bound of [4] for the 2-user case. The bound is valid for any memoryless channel and any number of users

2) We derive the sum-capacity for the symmetric K-user LDC exactly; the outer bound was obtained in [2] and here we show achievability. Interestingly, the scheme only requires one user to be fully cognitive of the messages of all other users, which considerably relaxes the CMS requirement.

3) We derive the sum-capacity for the symmetric K-user Gaussian noise channel to within a constant additive and multiplicative gap. The additive gap is a function of the number of users and grows as (K – 2) log (K – 2); the proposed achievable scheme can be thought of as a MIMO- broadcast scheme

4) Achievability scheme only requires the first K – 1 cognitive users to know the message of the primary user but not those of other cognitive users

General Outer-bound

$$R_i \leq I(Y_i; X_{[i:K]} | X_{[1:i-1]}),$$

$$\sum_{j=i}^{K} R_j \leq \sum_{j=i}^{K} I(Y_j; X_{[j:K]} | X_{[1:j-1]}, Y_{[1:j-1]}),$$

$$i \in [1:K]$$

Linear Deterministic Channel

Linear Deterministic Channel Model (LDC):

$$Y_\ell = \sum_{i \in [1:K]} \mathbf{S}^{m-n_{\ell i}} X_i, \ \ell \in [1:K],$$

$$m := \max_{(i,j) \in [1:K]^2} \{n_{ij}\}$$

Sum-rate outer-bound:

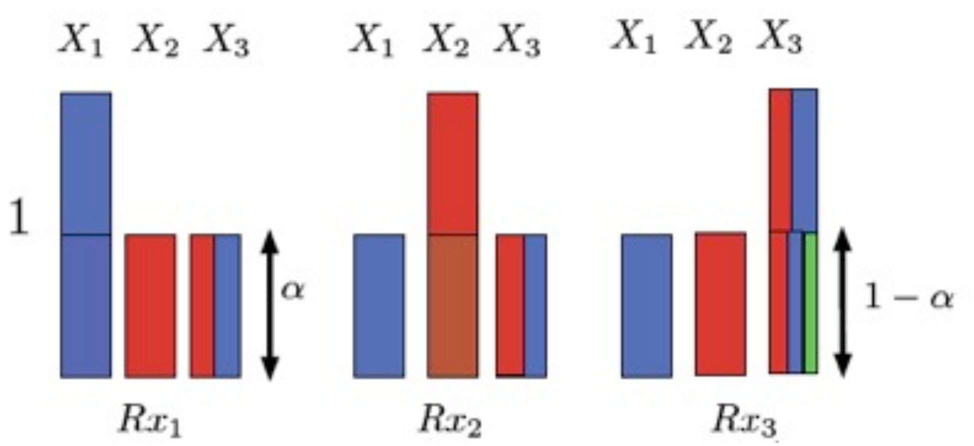
$$\begin{aligned}
\sum_{k=1}^{K} R_k &\leq \sum_{k=1}^{K} H\Big(Y_k | X_1, \ldots, X_{k-1}, Y_1, \ldots, Y_{k-1}\Big) \\
&= \sum_{k=1}^{K-1} H\Big(\mathbf{S}^{m-n_D} X_k + \mathbf{S}^{m-n_I} (\sum_{i=k+1}^{K} X_i) + \mathbf{S}^{m-n_I} (\sum_{i=1}^{K-1} X_i) \\
&\qquad |X_1, \ldots, X_{k-1}, \mathbf{S}^{m-n_I} (\sum_{i=k}^{K} X_i)\Big) \\
&\quad + H\Big(\mathbf{S}^{m-n_D} X_K | \mathbf{S}^{m-n_I} X_K\Big) \\
&\leq \sum_{k=1}^{K-1} H\Big((\mathbf{S}^{m-n_D} + \mathbf{S}^{m-n_I}) X_k\Big) + H\Big(\mathbf{S}^{m-n_D} X_K | \mathbf{S}^{m-n_I} X_K\Big) \\
&\leq n_S (K-1) \max\{1, \alpha\} + n_S [1-\alpha]^+ \\
&\leq n_S \Big(K \max\{1, \alpha\} - \alpha\Big).
\end{aligned}$$

Linear Deterministic Channel (Cont'd)

$$X_j = U_j, \ j \in [1:K-1],$$

$$X_K = \begin{bmatrix} I_{n_c} & 0_{n_c \times [n_d - n_c]^+} \\ 0_{[n_d - n_c]^+ \times n_c} & 0_{[n_d - n_c]^+ \times [n_d - n_c]^+} \end{bmatrix} \left(\sum_{j=1}^{K-1} U_j \right) + \begin{bmatrix} 0_{n_c \times n_c} & 0_{n_c \times [n_d - n_c]^+} \\ 0_{[n_d - n_c]^+ \times n_c} & I_{[n_d - n_c]^+} \end{bmatrix} U_K,$$

Example 3 User LDC:



Gaussian Channel

Gaussian Channel Model (LDC):

$$Y_\ell = \sum_{i \in [1:K]} h_{\ell i} X_i + Z_\ell, \ \ell \in [1:K],$$

Sum-rate outer-bound:

$$\begin{aligned}
\sum_{k=1}^{K} R_k &\leq \log\left(1 + \left(|h_\mathrm{d}| + (K-1)|h_\mathrm{i}|\right)^2\right) \\
&+ (K-2)\log(2) + (K-2)\log\left(1 + \frac{\big||h_\mathrm{d}| - h_\mathrm{i}\big|^2}{2}\right) \\
&+ \log\left(1 + \frac{|h_\mathrm{d}|^2}{1 + (K-1)|h_\mathrm{i}|^2}\right)
\end{aligned}$$

Sum-rate inner-bound: *MIMO BROADCAST DIRTY PAPER CODING*

$$R_1 = \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}| \sum_{j=2}^{K} \alpha_j\right|^2}{1 + |h_\mathrm{i}|^2 \sum_{k=2}^{K} |\gamma_k|^2}\right)$$

$$R_j = \log\left(1 + \frac{\big||h_\mathrm{d}| - h_\mathrm{i}\big|^2 |\beta|^2 + |h_\mathrm{d}|^2 |\gamma_j|^2}{1 + |h_\mathrm{i}|^2 \sum_{k=j+1}^{K} |\gamma_k|^2}\right),$$

$$R_K = \log\left(1 + |h_\mathrm{d}|^2 |\gamma_K|^2\right)$$

Constant Gap Results

By taking the difference between achievability scheme and outer bound

$$\mathrm{GAP} \leq (K-2)\log(K-2) + \log(2\exp(2)),$$

Conclusion

- General outer bound region for the K- user cognitive interference channel with cumulative message sharing. This computable outer bound was used to show that the symmetric sum-capacity is exactly achievable for the linear deterministic channel and to within constant gap for the Gaussian noise channel
- Interestingly the *same* sum-capacity outer bound may be achieved with a different message sharing structure with less message knowledge at the cognitive transmitters.

References

[1] K. Nagananda and C. Murthy, " Information theortic results for three user cognitive channels," in Global Telecommunications Conference 2009. GLOBECOME 2009. IEEE. IEEE, 2000, pp.1-6

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[3] A. Jafarian and S. Vishwanath, "On the capacity of multi-user cognitive radio networks," in *Proc. IEEE Int. Symp. Inf. Theory.* IEEE, 2009, pp. 601–605

[4] S. Rini, D. Tuninetti, and N.Devroye, " New inner and outer bounds for the discrete memoryless cognitive interference channel and some new capacity results," IEEE Trans. Theory, vol. 57, no. 7, pp 4087-4109, Jul. 2011