Channel Capacity under Sub-Nyquist Nonuniform Sampling

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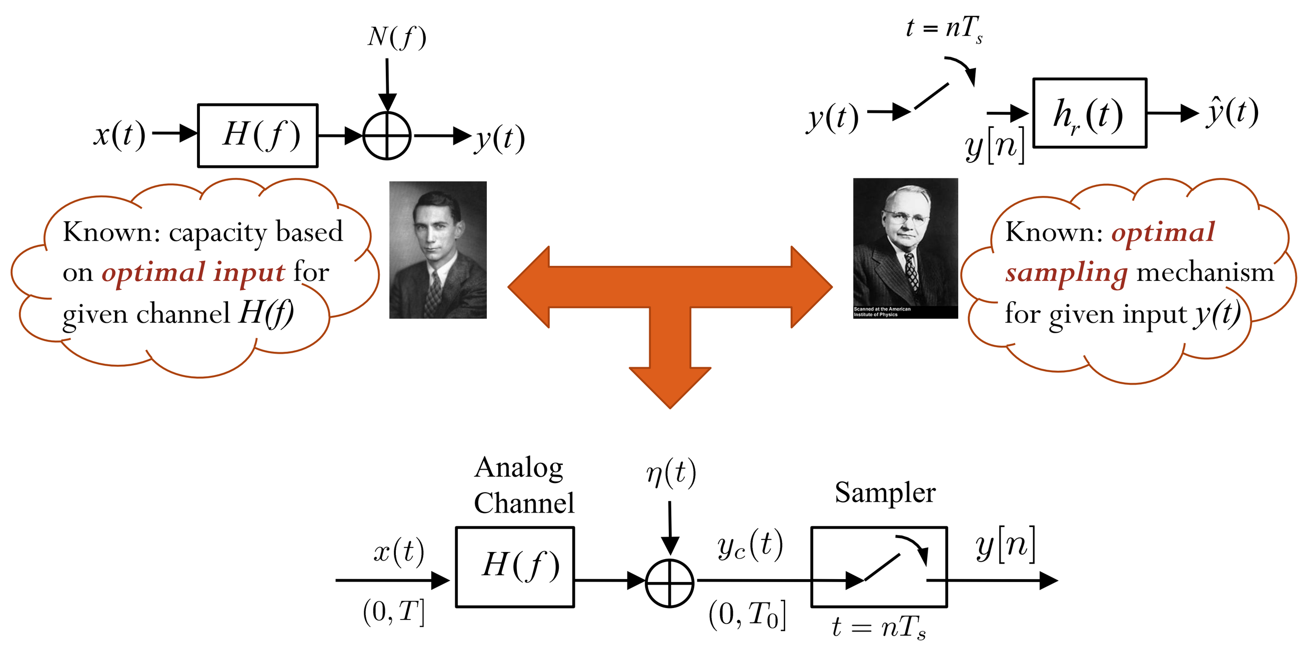
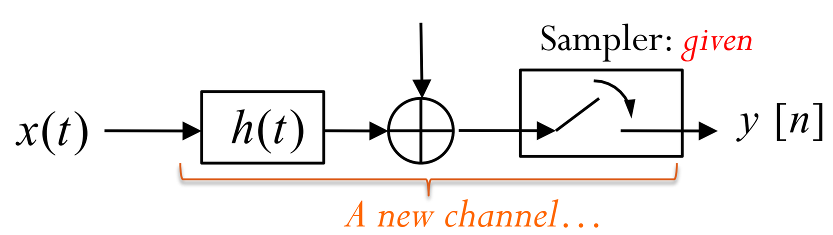
Background

- **Information Theory Meets Sampling Theory**

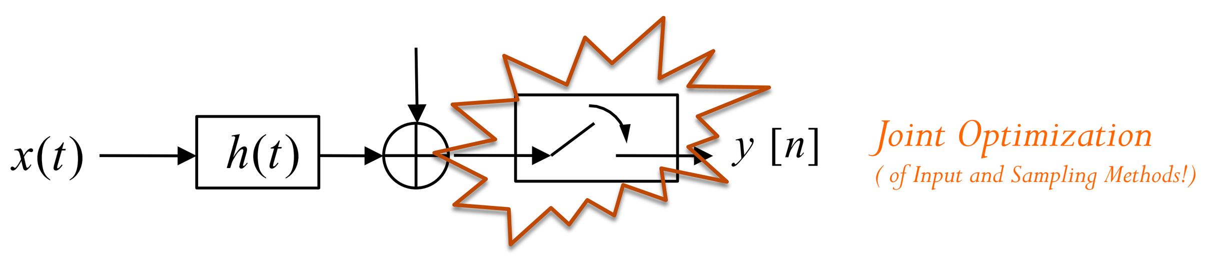


– How to jointly optimize the input distribution and sampling methods?

- **What is Sampled Channel Capacity [ChenEldarGoldsmith'2011]**
  – For a *given* sampling system:

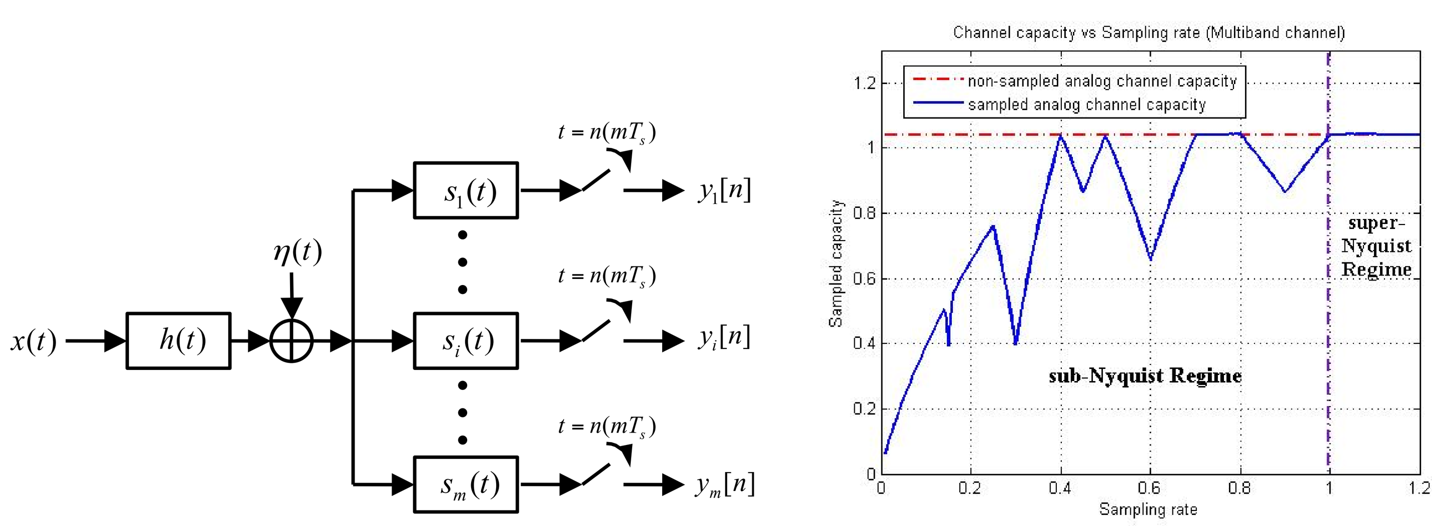


– For a large *class* of sampling systems



Motivation

- Consider a bank of filters each followed by a uniform sampler…

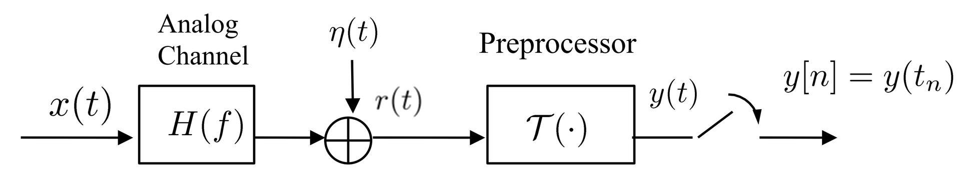


– The sampled capacity is *nonmonotonic* in the sampling rate

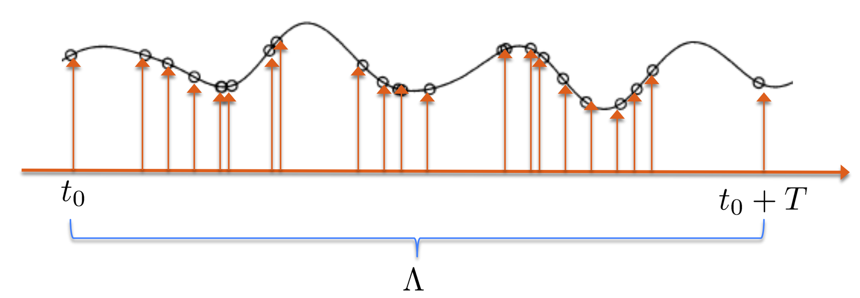
Questions

- Will more general nonuniform sampling methods improve capacity?
- Which sampling systems can maximize capacity for a given sampling rate?
- What is the gap between sub-Nyquist sampled capacity and analog capacity?

Problem Formulation



- **Sampling Rate**
  – Beurling Density: $f_s = \lim_{T\to\infty} \inf_{t_0} \frac{\#\Lambda\cap[t_0,t_0+T]}{T}$



– Time-preserving Preprocessing System: A system that preserves the time scales
  – *Counterexample:* $\mathcal{T}(x(t)) = x(2t)$

- **Sampled Channel Capacity (Perfect Channel State Information at Both Sides)**

– For a given system $\mathcal{P}$:

$$C^{\mathcal{P}}(f_s) = \lim_{T\to\infty} \inf_{p(x)} \sup \frac{1}{2T} I\left(x([-T,T]), \{y(t_n)\}_{[-T,T]}\right)$$

– For the class of time-preserving systems: $C(f_s) = \lim \sup_{\mathcal{P}} C^{\mathcal{P}}(f_s)$

Converse: (Main Result)

- **Theorem 1.** Consider any time-preserving sampling system with rate f_s. Suppose that there exists a frequency set $B_{\rm m}$ that satisfies $\mu(B_{\rm m}) = f_s$ and

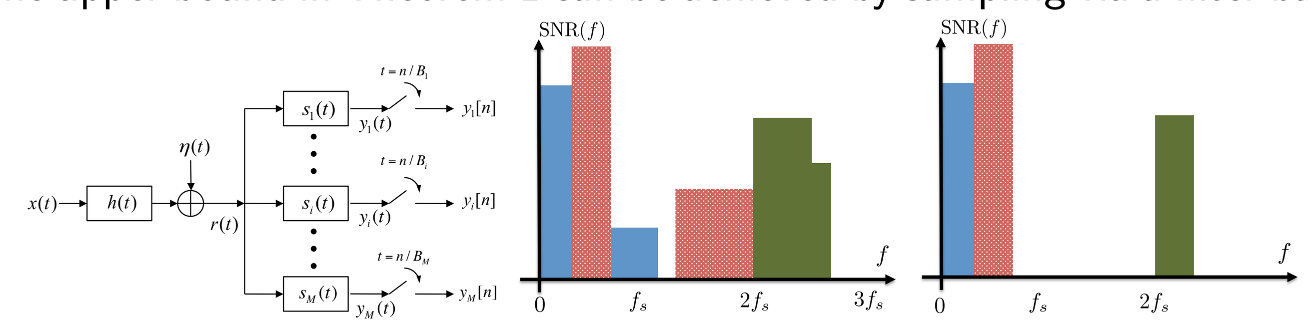
$$\int_{f\in B_{\rm m}} \frac{|H(f)|^2}{\mathcal{S}_\eta(f)} {\rm d}f = \sup_{B:\mu(B)=f_s} \int_{f\in B} \frac{|H(f)|^2}{\mathcal{S}_\eta(f)} {\rm d}f.$$

Then the sampled channel capacity can be upper bounded by

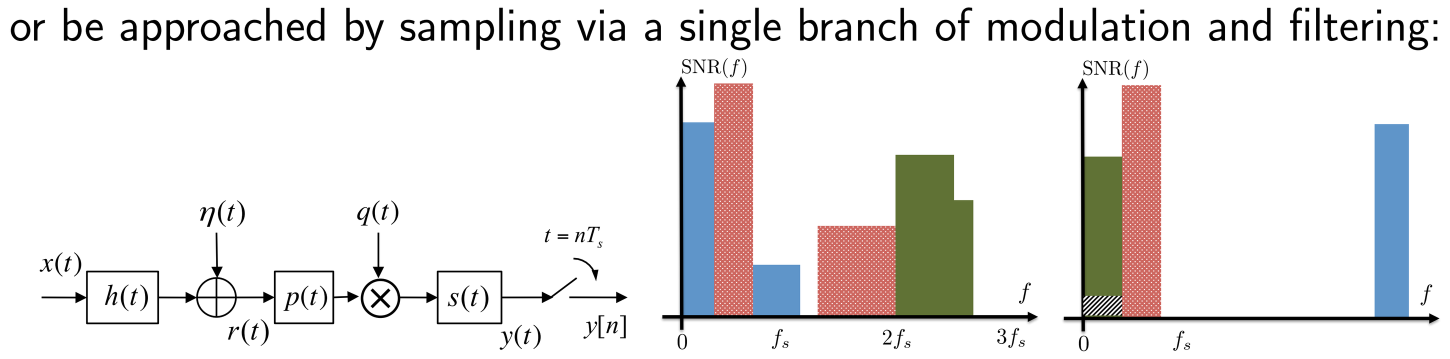
$$C_{\rm u}(f_s, P) = \int_{f\in B_{\rm m}} \frac{1}{2}\left[\log\left(v\frac{|H(f)|^2}{\mathcal{S}_\eta(f)}\right)\right]^+ {\rm d}f,$$

Achievability

- The upper bound in Theorem 1 can be achieved by sampling via a filter bank:



or be approached by sampling via a single branch of modulation and filtering:



Implications

- **The Optimal Sampling Method:**

  – extracts out a frequency set with the highest SNR

  – suppresses aliasing

  – results in a capacity monotonic in the sampling rate

- **Irregular nonuniform sampling grid does not improve capacity.**

  – robust to mild permutation of the sampling grid

- **When the sampling rate is increased, the adjustment of the sampling hardware for lter-bank sampling is incremental.**

The Way Ahead

- **If the CSI is not perfectly known or if the channel state can be varying:**
  – Alias-suppressing sampling is not necessarily optimal.
  – May need to scramble spectral contents.
  – May need different objective metrics (e.g. minimaxity).

- **Decoding-constrained information theory:**
  – Sampling systems can be viewed as part of the decoding method.
  – How to find the capacity-achieving input and decoding strategy if the decoding strategy needs to be picked from a given set (possibly infinitely many choices)