Lossy joint source-channel coding in the finite blocklength regime

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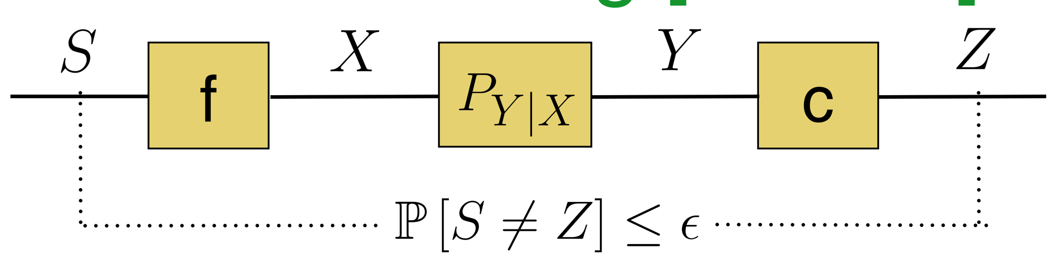
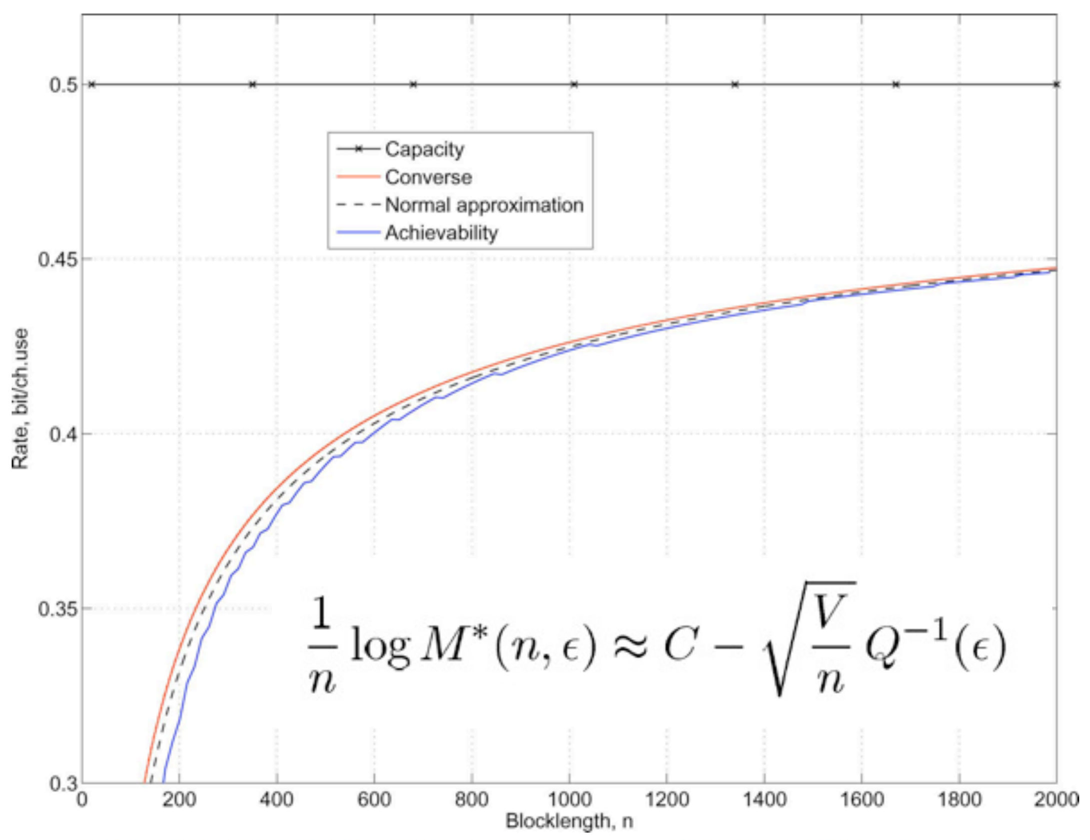
Channel coding [PPV10]



S equiprobable on $\{1, \dots, M\}$

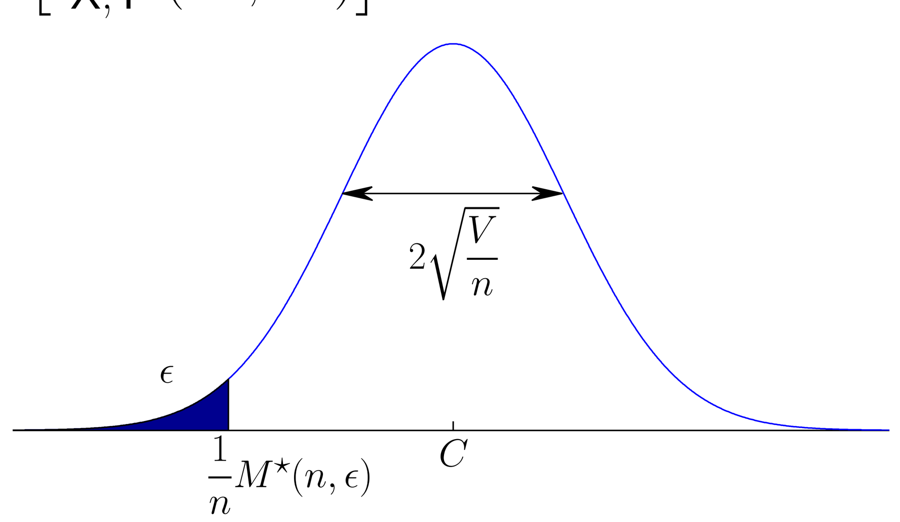
Channel block coding: $X \to X^n$, $Y \to Y^n$



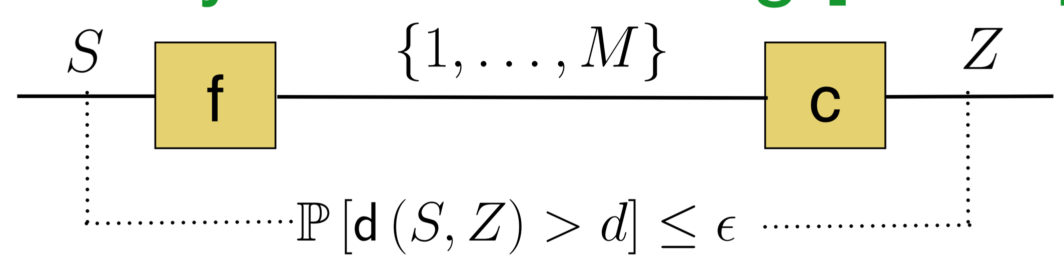
Rate-blocklength tradeoff for the BEC with erasure probability $\delta = 0.5$ and maximal block error rate $\epsilon = 10^{-3}$.

$$C = \mathbb{E}\left[\imath_{X;Y^\star}(X^\star; Y^\star)\right] = \inf_{P_X} I(X;Y)$$

$$V = \mathrm{Var}\left[\imath_{X;Y^\star}(X^\star; Y^\star)\right]$$



Lossy source coding [KV12]



Source block coding: $S \to S^k$, $Z \to Z^k$

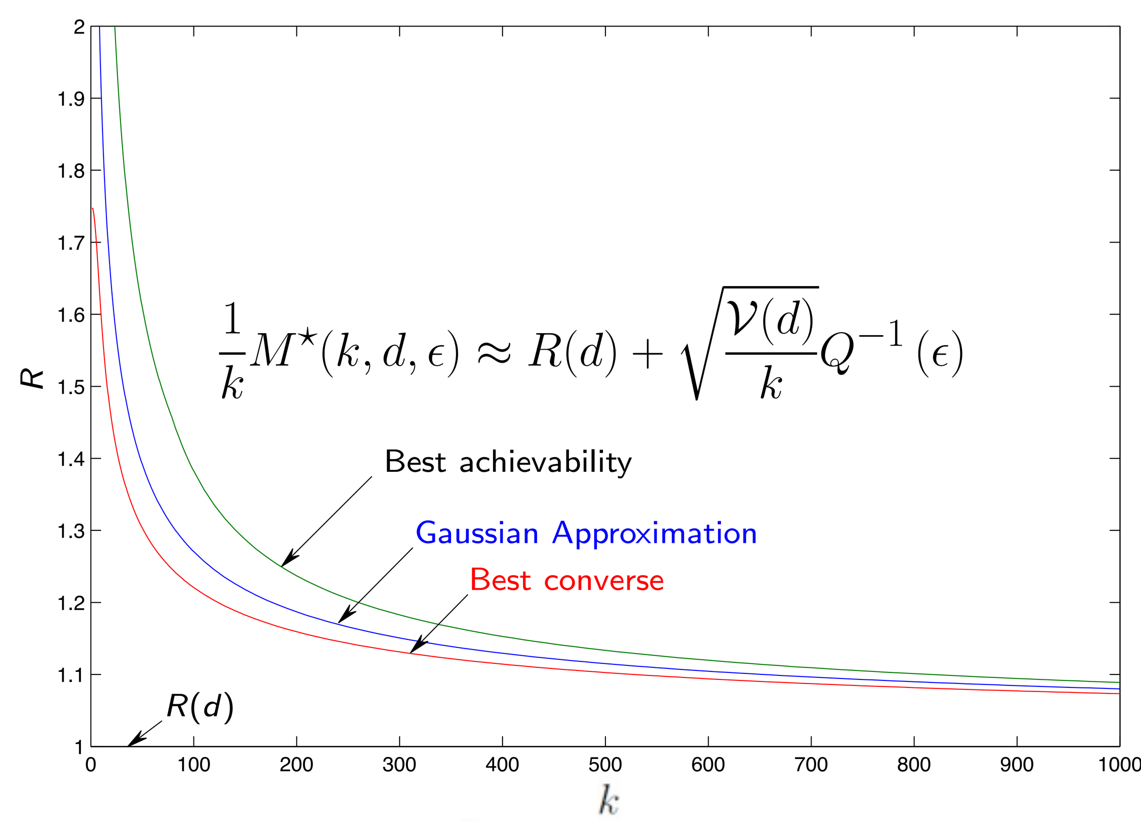
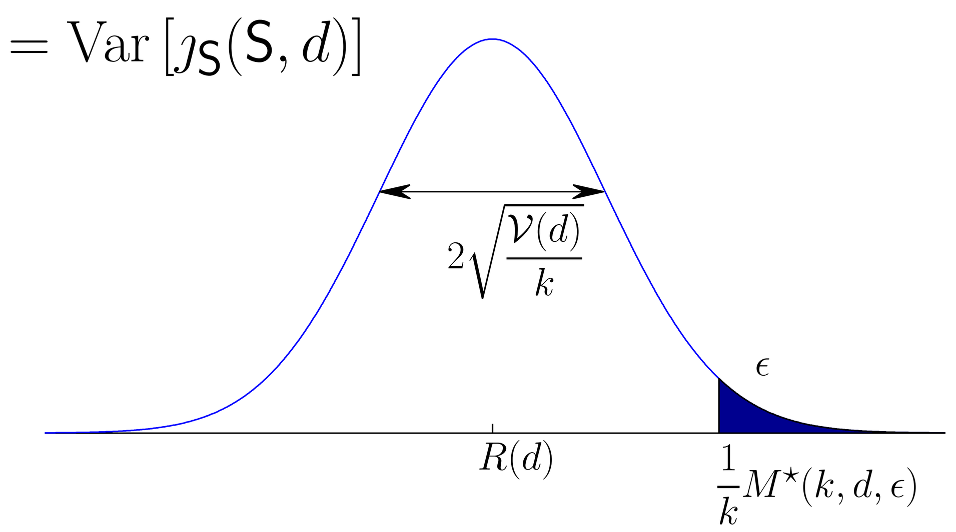


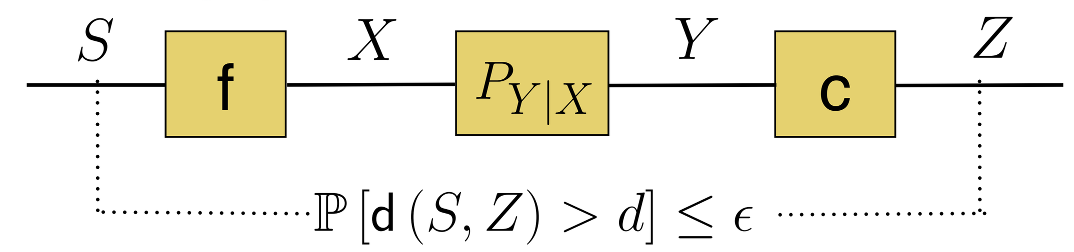
Figure: Bounds to $R(k, d, \epsilon)$ for the Gaussian i.i.d. source with mean-square error distortion, $d = \frac{1}{4}$, $\epsilon = 10^{-2}$.

$$R(d) = \mathbb{E}\left[\jmath_S(S, d)\right] = \inf_{\substack{P_{Z|S}:\\ \mathbb{E}[d(S,Z)] \le d}} I(S;Z)$$

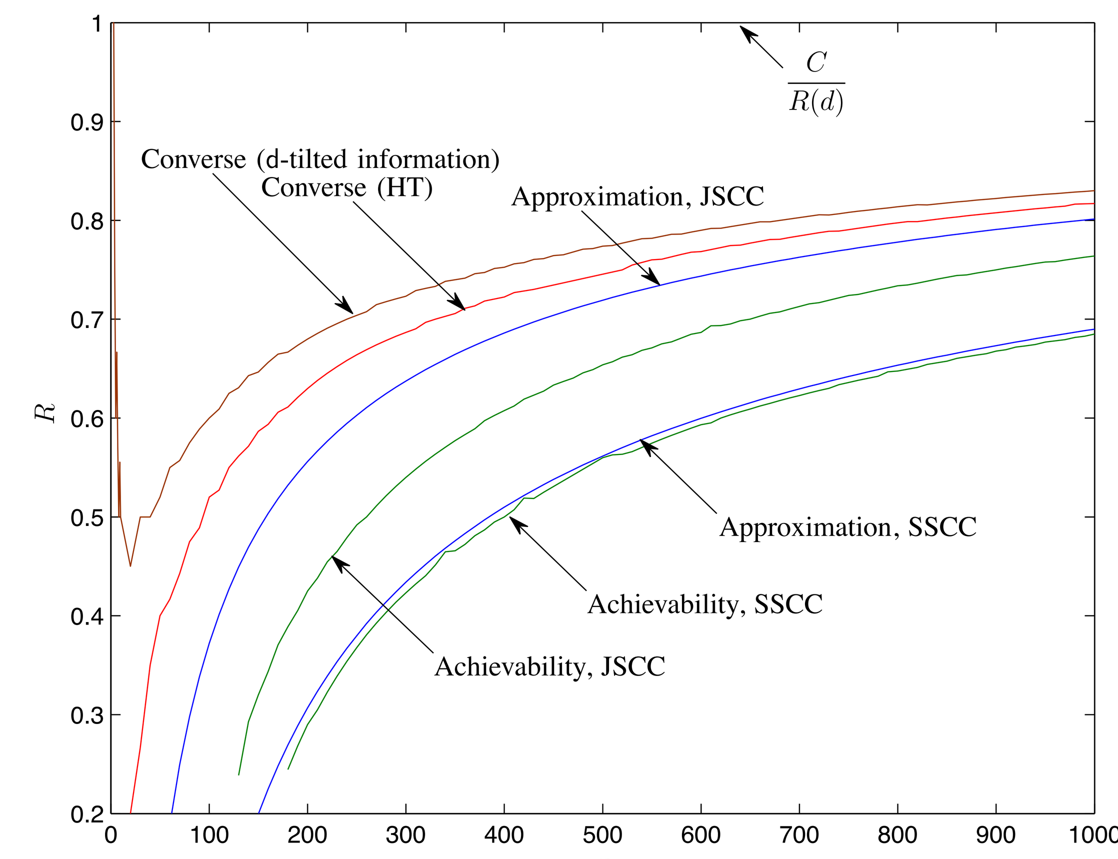
$$\mathcal{V}(d) = \mathrm{Var}\left[\jmath_S(S, d)\right]$$



Lossy joint source-channel coding

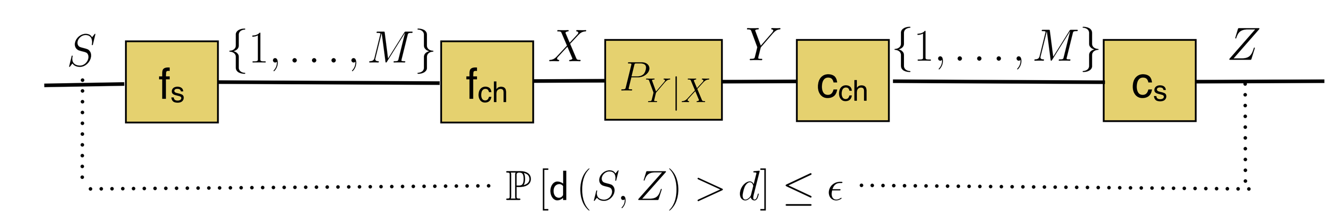


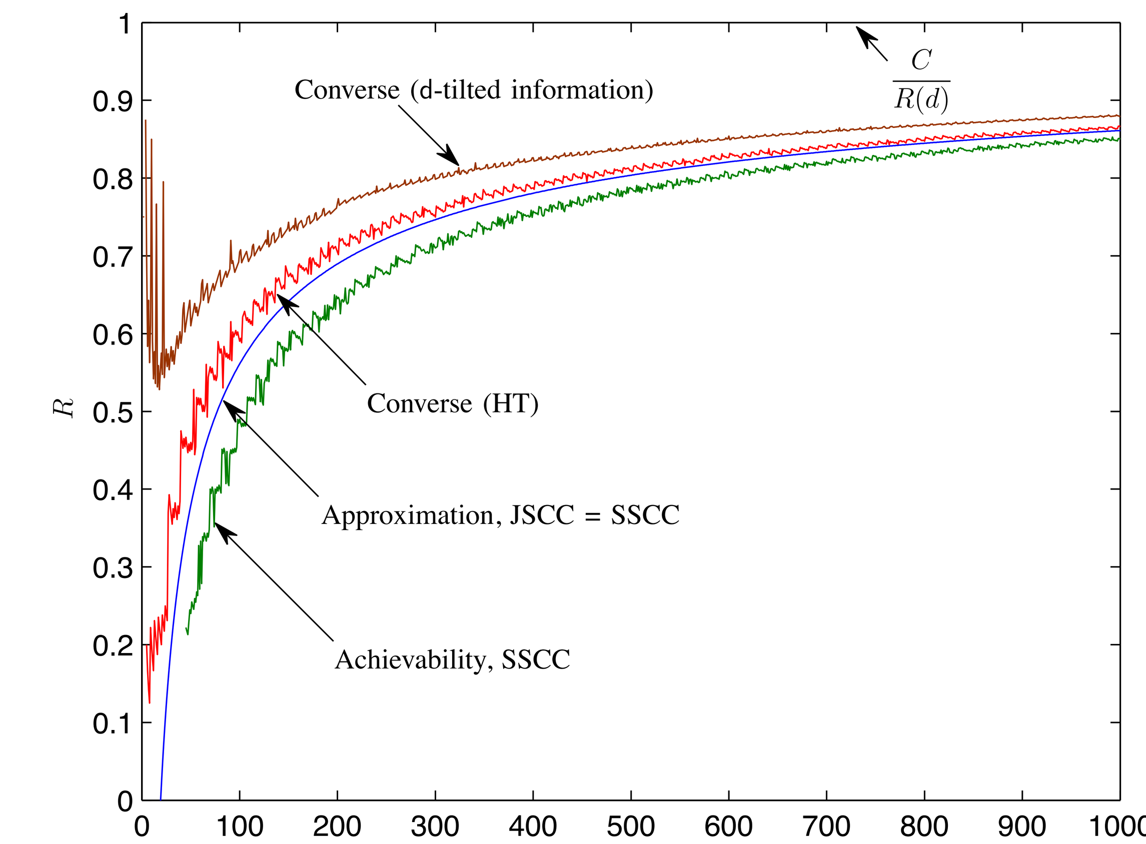
Block coding: $S \to S^k$, $X \to X^n$, $Y \to Y^n$, $Z \to Z^k$



Rate-blocklength tradeoff for the transmission of a GMS with $\frac{d}{\sigma^2} = 0.5$ over an AWGN channel with $P = 1$, $\epsilon = 10^{-2}$.

Separate source-channel coding:





Rate-blocklength tradeoff for the transmission of a fair BMS over a BSC with crossover probability $\delta = d = 0.11$ and $\epsilon = 10^{-2}$.

Joint source-channel coding:

$$nC - kR(d) \approx \sqrt{nV + k\mathcal{V}(d)}\,Q^{-1}(\epsilon)$$

Separate source-channel coding:

$$nC - kR(d) \approx \min_{\eta + \zeta \le \epsilon}\left\{\sqrt{nV}Q^{-1}(\eta) + \sqrt{k\mathcal{V}(d)}Q^{-1}(\zeta)\right\}$$

Converse

Any (d, ϵ) code for source S and channel $P_{Y|X}$ must satisfy

$$\epsilon \ge \sup_{\gamma > 0}\left\{\sup_{P_{\bar{Y}}} \mathbb{E}\left[\inf_{x \in \mathcal{X}} \mathbb{P}\left[\jmath_S(S,d) - \imath_{X;\bar{Y}}(x;Y) \ge \gamma \mid X = x, S\right]\right] - \exp(-\gamma)\right\}$$

Achievability

There exists a (d, ϵ) source-channel code with

$$\epsilon \le \inf_{P_Z, P_X, M, \gamma > 0}\left\{\mathbb{E}[\exp\{-|\imath_{X;Y}(X;Y) + \log P_Z(B_d(S)) - \log H_M - \log\gamma|^+\}] + \mathbb{E}\left[(1 - P_Z(B_d(S)))^M\right] + e^{-\gamma}\right\}$$

- the expectations are with respect to $P_S P_Z P_X P_{Y|X}$
- $H_M = \sum_{m=1}^{M}\frac{1}{m}$
- $B_d(s) = \{z \in \hat{\mathcal{M}} \colon \mathsf{d}(s,z) \le d\}$

information density: $\imath_{X;Y}(x;y) = \log \frac{dP_{Y|X=x}(y)}{dP_Y(y)}$

d-tilted information:

$$\jmath_S(s, d) = \imath_{S;Z^\star}(s;z) + \lambda^\star d(s,z) - \lambda^\star d \qquad \text{for } P_Z^\star\text{-a.e. } z$$

$$= \log \frac{1}{\mathbb{E}\left[\exp\{\lambda^\star d - \lambda^\star \mathsf{d}(s, Z^\star)\}\right]} \qquad (\mathbb{E} \text{ is wrt unconditional } P_Z^\star)$$

– $Z^\star$ is the rate-distortion-achieving random variable;

– $\lambda^\star = -R'(d)$.

References

[PPV10] Y. Polyanskiy, H. V. Poor, and S. Verdú. Channel coding rate in finite blocklength regime. *IEEE Transactions on Information Theory*, 56(5):2307–2359, May 2010.

[KV12] V. Kostina and S. Verdú. Fixed-length lossy compression in the finite blocklength regime. *IEEE Transactions on Information Theory*, 58(6):3309–3338, June 2012.

[TVG11] A. Tauste Campo, G. Vazquez-Vilar, A. Guillén i Fàbregas, and A. Martinez. Random-coding joint source-channel bounds. In *ISIT*, Saint-Petersburg, Russia, Aug. 2011.

[WIK11] D. Wang, A. Ingber, and Y. Kochman. The dispersion of joint source-channel coding. In *49th Allerton Conference*, Monticello, IL, Sep. 2011.